CFD analysis of single-phase flows inside helically coiled tubes

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\textbf{A B S T R A C T}

It has been well established that heat transfer in a helical coil is higher than that in a corresponding straight pipe. However, the detailed characteristics of fluid flow and heat transfer inside helical coil is not available from the present literature. This paper brings out clearly the variation of local Nusselt number along the length and circumference at the wall of a helical pipe. Movement of fluid particles in a helical pipe has been traced. CFD simulations are carried out for vertically oriented helical coils by varying coil parameters such as (i) pitch circle diameter, (ii) tube pitch and (iii) pipe diameter and their influence on heat transfer has been studied. After establishing influence of these parameters, correlations for prediction of Nusselt number has been developed. A correlation to predict the local values of Nusselt number as a function of angular location of the point is also presented.

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1. Introduction

It has been widely reported in literature that heat transfer rates in helical coils are higher as compared to that in straight tubes. Due to the compact structure and high heat transfer coefficient, helical coil heat exchangers are widely used in industrial applications such as power generation, nuclear industry, process plants, heat recovery systems, refrigeration, food industry, etc. (Abdulla, 1994; Bai, Guo, Feng, & Chen, 1999; Futagami & Aoyama, 1988; Jensen & Bergles, 1981; Patankar, Pratap, & Spalding, 1974; Xin, Awwad, Dong, & Ebadin, 1996).

Heat exchanger with helical coils is used for residual heat removal systems in islanded or barge mounted nuclear reactor system, wherein nuclear energy is utilised for desalination of seawater (Manna, Jayakumar, & Grover, 1996). The performance of the residual heat removal system, which uses a helically coiled heat exchanger, for various process parameters was investigated by Jayakumar and Grover (1997). The work had been extended to find out the stability of operation of such a system when the barge on which it is mounted is moving (Jayakumar, Grover, & Arakeri, 2002). In all these studies, empirical correlations were used to estimate the amount of heat transfer and pressure drop in the helical coils.

1.1. Characteristics of helical coil

In the present analysis, we consider helical coils which are vertically oriented, i.e., where the coil axis is vertical. Fig. 1 gives the schematic of the helical coil. The pipe has an inner diameter 2\(r\). The coil diameter (measured between the centres of the pipes) is represented by 2\(r_c\). The distance between two adjacent turns, is called pitch, \(H\). The coil diameter is also called as pitch circle diameter (PCD). The ratio of pipe diameter to coil diameter (\(r/r_c\)) is called curvature ratio, \(\alpha\). The ratio of pitch to developed length of one turn (\(H/2\pi r_c\)) is termed non-dimensional pitch, \(\lambda\). Consider the projection of the coil on a plane passing through the axis of the coil. The angle, which projection of one turn of the coil makes with a plane perpendicular to the axis, is called the helix angle, \(\alpha\). For any cross-section of the pipe, created by a plane passing through the coil axis, the side of pipe wall nearest to the coil axis is termed inner side and the farthest side is termed as outer side.

Similar to Reynolds number for flow in pipes, Dean number is used to characterise the flow in a helical pipe. The Dean number, \(De\) is defined as,

\[
De = Re \sqrt{\frac{r}{r_c}},
\]  

where, \(Re\) is the Reynolds number, \(\frac{2\pi r_c \rho \nu}{\mu}\).

Many researchers have identified that a complex flow pattern exists inside a helical pipe due to which the enhancement in heat transfer is obtained. The curvature of the coil governs the centrifugal...
gal force while the pitch (or helix angle) influences the torsion to which the fluid is subjected to. The centrifugal force results in development of secondary flow (Darvid, Smith, Merrill, & Brain, 1971). Due to the curvature effect, the fluid streams in the outer side of the pipe moves faster than the fluid streams in the inner side of the pipe. The difference in velocity sets-in secondary flows, whose pattern changes with the Dean number of the flow.

1.2. Critical Reynolds number

Transition from laminar to turbulent flow regime takes place at a Reynolds number higher than that for a straight pipe. Critical Reynolds number ($Re_{cr}$) can be estimated using the correlations developed by Ito (1959), Schmidt (1967), Srinivasan, Nandapurkar, and Holland (1970) or Janssen and Hoogendoorn (1978). A plot of $Re_{cr}$ for the curvature ratio from 0.01 to 0.25 is given in Fig. 2. In the lower range of curvature ratios ($\delta < 0.05$), all of the correlations provide approximately the same value for the $Re_{cr}$. Correlations provided by Ito and Schmidt give almost equal values of $Re_{cr}$ for the entire range of curvature ratios which is of practical interest and these correlations are used in the present work.

1.3. Review of heat transfer in turbulent flows

Heat transfer and flow through a curved tube is comprehensively reviewed first by Berger, Talbot, and Yao (1983) and subsequently by Shah and Joshi (1987). The latest review of flow and heat transfer characteristics is provided by Naphon and Wongwises (2006). Most of the investigations on heat transfer coefficients are for the boundary conditions such as constant wall temperature or constant heat flux (Nandakumar & Masliyah, 1982; Prabhanjan, Rennie, & Raghavan, 2004; Shah & Joshi, 1987). The situation of constant wall temperature is idealised in heat exchangers with phase change such as condensers. Modelling of constant wall heat flux boundary condition find applications in dealing with heat transfer studies of nuclear fuel elements and electrically heated tubes.

Heat transfer in helical coils has been experimentally investigated by Seban and McLaughlin (1963) both for laminar and turbulent flow regimes for flow of water for constant wall flux BC. The range of Reynolds number studied was 6000–65,500 and the Prandtl number variation was from 2.9 to 5.7. The curvature ratios of the coils were 0.0096 and 0.0588. In the development of the correlation, data reduction was done assuming constant fluid properties and considering the tubes to be straight ones. The authors had stated that these assumptions may lead to 10% error in the calculated values.

Rogers and Mayhew (1964) studied heat transfer to fluid flowing inside a helical pipe which was heated by steam. The correlation is valid for a range of Reynolds number from 10,000 to 100,000. Curvature ratios of the coils used in studies were 0.0926, 0.075 and 0.05. Fluid properties were estimated at bulk mean temperature.

Mori and Nakayama (1967a) investigated forced convective heat transfer in turbulent regime for wall heat flux boundary condition. Variation of physical properties with temperature changes were not taken into account in their work. They have considered two configurations of helical coils, viz., coils with curvature ratio of 0.0535.
and 0.025. Subsequently Mori and Nakayama (1967b) studied heat transfer under constant wall temperature boundary condition for the same helical coils. They observed that the Nusselt number is remarkably affected by a secondary flow due to curvature. They stated that the same formula, which was used for estimation of Nusselt number for wall heat flux boundary condition, could be used for the wall temperature boundary condition as well.

Heat transfer and pressure drop in helical pipes was studied by Yildiz, Bicer, and Pehlivan (1997). They have studied both empty tube and with heat transfer enhancement. Fully developed turbulent forced convective heat transfer in a helical coil, which has a substantial coil pitch was numerically studied by Yang and Ebadian (1996). Standard \( k-\varepsilon \) model with the constants recommended by Launder and Spalding (1972) were used in this study. They did not consider any physical property variation and no generalised correlation was developed for estimation of heat transfer coefficients. Lin and Ebadian (1997) applied the same turbulence model to study the developing turbulent heat transfer in a helical coil of definite pitch for a constant wall temperature case. The FLUENT/UNS code was used as the numerical solver. Subsequently the effect of intensity of inlet turbulence on heat transfer rates was studied by Lin and Ebadian (1999) using the same numerical solver, FLUENT/UNS.

Guo, Chen, Feng, and Bai (1998) has developed correlations for estimation of Nusselt number for steady state and pulsating turbulent flow through helical coils in the range of Reynolds number from 6,000 to 180,000. However this correlation does not include any coil parameters (such as curvature ratio) and is applicable only to their setup.

Li, Lin, and Ebadian (1998) numerically investigated turbulent mixed convective heat transfer in the entrance region of curved pipes for constant wall temperature boundary condition using FLUENT/UNS code. No correlation for prediction of heat transfer rates is provided by the authors. An experimental study of turbulent heat transfer in a horizontally coiled helical tube was done by Bai et al. (1999), where the working fluid was heated by resistance heating of the tube wall by passage of alternating current. However, the correlation is not expressed in terms of Dean number as they have investigated heat transfer in only one coil. From their results, it is observed that the average Nusselt number for a horizontally oriented helical tube is less than that for a vertically oriented one for the same conditions.

Chagny, Castelain, and Peerhoossaini (2000) studied the chaotic heat transfer in heat exchanger designs and compared them with regular regimes for a range of Reynolds numbers from 30 to 30,000 and for a number of Prandtl numbers. Pressure drop and heat transfer in tube-in-tube helical heat exchanger under turbulent flow conditions was studied by Kumar, Saini, Sharma and Nigam (2006) using the CFD package FLUENT 6. They have used the Wilson plot method (Wilson, 1915) for data reduction. However, no correlation for estimation of Nu was given in the paper.

Recently Jayakumar, Mahajani, Mandal, Vijayan, and Bhoi (2008) have developed a correlation for estimation of inside heat transfer coefficient for flow of single-phase water through helically coiled heat exchangers. The correlation, which is validated against experiments, is applicable to a specific configuration of helical coils. Also the results presented in the paper describe only the overall heat transfer coefficient. The local variation of the Nusselt number was not reported.

It can be seen that influence of various coil parameters are not thoroughly studied numerically so as to enable the generation of a correlation for prediction of Nusselt number. Also the ranges of Reynolds number and curvature ratio in the previous studies did not cover the whole of the possible operating ranges. The investigations reported for estimation of Nusselt number were found to be carried out with constant fluid properties. Also the published literature lacks much detail in the finer aspects of local heat transfer rates and the fluid flow characteristics inside the helical coils. In the present work, variations of Nusselt number along the circumference of the pipe and along the length of the coil have been presented. Much insight has been brought out regarding the nature of fluid flow inside a helical pipe.

In the next section, characteristics of fluid flow and heat transfer in a helical pipe are presented. Helical coils of 12 different configurations have been analysed and the dependency of coil parameters on local and average Nusselt number is brought out. Using results of these computations, where temperature dependant fluid properties are used, correlations for estimation of inside heat transfer coefficient for flow of single-phase water through helical coils are developed. Further, variation of Nusselt number around the periphery at a given pipe cross-section is analysed and a correlation for prediction of local Nusselt number as a function of average Nusselt number and angular position is presented.

2. Nature of turbulent flow and heat transfer in helical coils

Analysis with heat transfer to water flowing through a helical coil is carried out using CFD package FLUENT version 6.3. As a representative case, coil of PCD = 200 mm and coil pitch of 30 mm is presented for discussion. Diameter of the pipe used in the coil is 20 mm. The geometry and the mesh were created using GAMBIT 2.3 of the FLUENT package starting from its primitives. Boundary layer mesh was generated for the pipe fluid volume. The Grid used for this analysis is given in Fig. 3. This is the optimised grid after the grid independency studies which has been detailed out in Jayakumar et al. (2008). The mesh density was changed from 3.5563e8 cells m$^{-3}$ to 12.365e8 cells m$^{-3}$. It is found that refinement after a mesh density of 9.963e8 cells m$^{-3}$ does not decrease the energy and mass errors in any appreciable way and mesh with this density is chosen for analysis.

Pressure velocity coupling was done using the SIMPLEx scheme. Momentum equations were discretised using QUICK scheme. The

Fig. 3. (a) Grid of the helical pipe used for analysis. (b) Grid at any cross-section of the helical pipe.
realizable $k-\varepsilon$ turbulence model (Shih, Liu, Shabbir, Yang, & Zhu, 1995) is used in these computations. This scheme is ideal for flows involving rotation, boundary layers under strong adverse pressure gradients, separation, and recirculation (Kim et al., 1997; Shih et al., 1995; Fluent, 2007).

Power law scheme of discretisation is used for turbulent kinetic energy and dissipation rate equations. Convergence criterion used was 1.0e–5 for continuity, velocities, $k$, and $\varepsilon$. Temperature dependent properties as polynomial functions were used for water. Details of the modelling equations are available in Jayakumar et al. (2008). For the energy equation third order QUICK discretisation scheme was employed. Convergence criterion for energy balance was 1.0e–07.

The results of simulation are exported as a CGNS (CFD General Notation System, http://www.cgns.org) file from Fluent. The fields exported are pressure, temperature, velocity magnitude, axial, radial and tangential velocity, wall temperature, wall heat flux, density, specific heat and thermal conductivity.

For post-processing a visualisation package AnuVi developed by Computer Division, BARC is used. AnuVi is a cross-platform CFD post-processor and scientific visualisation framework and is built on top of the open source software like Python (http://www.python.org), Visualization Toolkit (VTK, http://www.vtk.org), WxWidgets (http://www.wxwidgets.org) and FFmpeg (http://www.ffmpeg.org). It can handle many standard file formats like CGNS, PLOT3D, VTK, STL, OBJ, BYU and PLY and has features to provide animation, extraction and derivation of data over many data components with advanced graphics (including shading, contouring, lighting and transparency). The package has features like Session Handling, Seamless Data Integration, Python Language Scripting, etc. Rendering is handled by OpenGL and can be accelerated with advanced graphics hardware. The feature of Python language scripting gives unlimited control to user which can be used for automation of data extraction and visualisation.

For extraction of data and visualisation, the CGNS files are processed to create planes at desired spacing in the computational domain. Since the fluid properties are temperature dependent, the bulk fluid temperature at a cross-section is evaluated using the relation,

$$T_b = \frac{\int u \rho C_p T dA}{\int u \rho C_p dA},$$

where $dA$ is an elemental area of the pipe cross-section (see Fig. 3b). The wall temperatures at four locations (inner, outer, top and bottom of the pipe) in a cross-section are also extracted. Using these data, values of local Nusselt number at four locations at that cross-section are calculated using the formula,

$$Nu_{loc} = \frac{2r}{k} \left( \frac{\bar{q}''}{T_w - T_b} \right),$$

where heat flux is calculated by, $\bar{q}'' = k(\partial T/\partial n)_w$; $n$ is the normal direction.

As used by Lin and Ebadian (1997), average $Nu$ at a cross-section may be estimated by,

$$Nu_{AV} = \frac{1}{2\pi} \int_0^{2\pi} \left( Nu_{\phi} \right) d\phi.$$

But this does not ensure that the Nusselt number so estimated is representative of the total heat flux in that cross-section. Hence, the mean Nusselt number is evaluated by;

$$Nu_{AV} = \frac{2r}{k_m} \left( \frac{\bar{q}''_{m}}{T_w, m - T_b} \right).$$

Here $k_m, T_{w, m}$ and $\bar{q}''_{m}$ are evaluated by the formula,

$$\bar{q}''_{m} = \frac{\int_0^{2\pi} (\phi \Delta A) d\phi}{\int_0^{2\pi} (\Delta A) d\phi} \cdot$$

where $\phi, k, T_w$ or $q''$ as the case may be and $\Delta A$ is an elemental area of a ring along the wall to which the parameter is associated to.

The above sets of operations are repeated at successive planes to cover the entire length of the pipe. All of the above processing have been done using Python scripts which runs on top of the AnuVi package. Various programs required to generate the cut planes, etc. was written in C++ programming language. MATLAB® has been extensively used for processing of the extracted data and regression analysis.

The remaining parts of this section describe the results of analysis carried out with constant wall temperature boundary condition and constant wall heat flux boundary condition.

### 2.1. Constant wall temperature boundary condition

In this analysis hot water at 330 K temperature and 0.8 m s$^{-1}$ velocity is entering the helical pipe at the top, where an inlet velocity boundary condition is specified. The flow velocity is such that the flow regime is turbulent. The fluid is made to cool down as it flows along the tube by specifying a wall temperature of 300 K. The fluid properties are estimated as per Eqs. (8a)–(8d). More details are available in Jayakumar et al. (2008).

The pipe wall, for the energy equation, a Dirichlet boundary condition and for momentum and pressure equations homogenous Neumann boundary condition are specified. At the inlet a turbulent intensity of 4% and hydraulic diameter of the largest size eddy, which is taken as 0.3 times pipe inner diameter, are specified. At the outlet, a pressure outlet boundary is enforced. Summary of the boundary conditions is given in Table 1.

$$\mu(T) = 2.1897e - 11T^4 - 3.055e - 8T^3 + 1.6028e - 5T^2 - 0.0037524T + 0.33158$$

(8a)

$$\rho(T) = -1.5629e - 5T^3 + 0.011778T^2 - 3.0726T + 1227.8$$

(8b)

$$k(T)=1.5362e - 8T^3 -2.261e - 0.5T^2 + 0.010879T - 1.0294$$

(8c)

$$C_p(T) = 1.1105e - 5T^3 - 0.0031078T^2 - 1.478T + 4631.9$$

(8d)

Fig. 4(a) shows an overview of velocity contours at various sections along the length of the coil, while the details at a few cross-sections are available in Fig. 4(b). The planes are identified by the angle $(\theta)$ which that plane makes with the plane passing through the pipe inlet. In Fig. 4(a), the first plane shown on the top is at 10° from the inlet (i.e., $\theta = 10°$) and the subsequent planes are 10° apart.

Up to an angle of $\theta = 35°$, the velocity profile at a cross-section is found to be symmetric. Subsequently, this uniform velocity pattern changes to a pattern with a high velocity region located at the outer side of the coil. This behaviour is seen predominantly by $\theta = 45°$ and continues to develop. It can be seen that by $\theta = 135°$, the high velocity region is present only in outer half cross-section. Area of high

<table>
<thead>
<tr>
<th>Field variable</th>
<th>Inlet</th>
<th>Walls</th>
<th>Outlet</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>0.8 m s$^{-1}$</td>
<td>0.0</td>
<td>$\partial u/\partial n = 0$</td>
</tr>
<tr>
<td>$v$</td>
<td>0.0</td>
<td>0.0</td>
<td>$\partial v/\partial n = 0$</td>
</tr>
<tr>
<td>$w$</td>
<td>0.0</td>
<td>0.0</td>
<td>$\partial w/\partial n = 0$</td>
</tr>
<tr>
<td>$T$</td>
<td>330 K</td>
<td>300 K</td>
<td>$\partial T/\partial n = 0$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>$\rho(T)$</td>
<td>$\rho(T)$</td>
<td>$\rho(T)$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$\phi(T)$</td>
<td>$\phi(T)$</td>
<td>$\phi(T)$</td>
</tr>
</tbody>
</table>
velocity region further reduces as the flow gets developed and covers approximately 1/3rd of the flow area by $\theta = 240^\circ$. No significant change in flow pattern is observed downstream.

Turbulent kinetic energy and turbulent intensity at various planes along the length of the coil are presented in Fig. 4(c and d) respectively. In the fully developed region, the turbulent kinetic energy has lower values at the inner side of the coil as compared to the values on the outer side. From Fig. 4(d) it can be noted that along the outer side of the coil the turbulent intensity attains a value as high as 13.4%.

Temperature distribution at various planes along the length of coil is shown in Fig. 5(a and b), the first one shows a global picture while the second one shows the details at selected planes. At the inlet, temperature is uniform across the cross-section. Since the wall is maintained at a lower temperature, the fluid cools down as it flows through the coil. Up to an angle of $20^\circ$, heat transfer is uniform along the periphery. In contrast to heat transfer in a straight tube, high temperature regions are seen on the outer side of the coil. This phenomena is predominant from the plane at angle $\theta = 50^\circ$. This trend continues to develop and by $\theta = 150^\circ$, clearly three regions viz., high temperature ($327-330 \text{ K}$) at the outer side of the coil, intermediate temperature ($321-324 \text{ K}$) at the centre and low temperature ($311-314 \text{ K}$) on the inner side of the coil, are visible. As the fluid flows down the pipe, this temperature profile gets developed and the area of high temperature region decreases and by $\theta = 360^\circ$, a fully developed temperature profile is attained and the fluid continues to loose heat due to the lower wall temperature.

As the fluid flows through the helical coil, fluid particles undergo rotational motion. The fluid particles also undergo movement from inner side of the coil to the outer side and vice versa. Fig. 6(a) shows particle trace for 10 fluid particles which are located along a line parallel to the $X$-axis at the pipe inlet. In Fig. 6(b), particle trace for a set of particles which were initially located parallel to the vertical axis is shown. It can be noted that these fluid particles are taking various trajectories and also move with different velocities. The particles, which were forming a line to begin with, have found to be totally scattered at the pipe exit. It can be clearly seen that the high velocity region oscillates as the fluid flows along the helical pipe. This causes fluctuations in the values of Nusselt number.

Fig. 7 further helps in better understanding of the flow characteristics in a helical coil. It shows the normalised time taken by few particles to traverse through a helical pipe. The particles whose ini-
Fig. 5. Temperature (K) contours at various planes along the length coil. (b) at various planes along the length coil.

Fig. 6. (a) Trace of fluid particles which are parallel to X-axis at the inlet. (b) Trace of fluid particles which are parallel to Z-axis at the inlet.

Fig. 7. Transit time of fluid particles in helical and straight pipes.
is shown in Fig. 9. In this figure, the planes are spaced 1° apart. The influence of helical nature of the pipe starts from 15° onwards. In the subsequent regions, there are periodic oscillations. Fig. 10(a–l) gives the variation of Nusselt number around the periphery at various cross-sections, (indicated by the angle $\theta$) along the length of the helical pipe. In each of the figures, 0° on the X-axis corresponds to inner side of the coil and the angle is measured in anti-clockwise direction. This means that 90° corresponds to bottom of the pipe; 180° corresponds to outer side of the pipe, etc. Only a few representative cross-sections, where significant change in nature of variation of $Nu$ is observed are presented. 

In the initial length of the pipe, up to an angle $\theta = 10^\circ$, marginally higher rates of heat transfer is observed at the upper side of the pipe. Due to gravity effect, the hotter fluid will be present at the top and this result in higher values of Nusselt number at that location. As the flow gets developed, when the effect of centrifugal forces becomes appreciable, region of higher heat transfer shifts from angle 270° to 180° i.e., form the upper side of the pipe to outer side of the coil. This shift gets completed by $\theta = 76^\circ$. It is observed that up to an angle of 140°, the percentage of circumference, which has a higher value of $Nu$ is predominant. This percentage decreases and by $\theta = 430^\circ$ onwards this region is so low that the average Nusselt number starts decreasing. Bai et al. (1999) has provided a figure showing ratio of local Nusselt number to average Nusselt number at various cross-sections along the length of the pipe are presented in Fig. 13. The trends observed are same as in Fig. 8. However, the local Nusselt numbers in this case are higher than the corresponding values for constant wall temperature case. Temperature profile is fully developed by $\theta = 360^\circ$.

The local values of Nusselt number at the inner, outer, top and bottom points at various cross-sections along the length of the pipe are presented in Fig. 12. These profiles show a much different scenario as compared to Fig. 5. In contrast to the constant wall temperature case, heat transfer is uniform around the periphery up to $\theta = 50^\circ$. In addition, the percentage length of region with higher heat transfer rates is comparatively high for the case of constant wall heat flux BC. Three distinct regions of heat transfer, similar to that observed at $\theta = 150^\circ$ for the wall temperature case, is found to be occurring almost at the same angle for the constant wall flux case also. However, the percentage of the periphery with high heat transfer rates is significantly higher for the wall flux case. This behaviour results in higher values of Nusselt numbers as compared to the previous case. Temperature profile is developed by $\theta = 360^\circ$.

Fig. 8. Variation of $Nu$ along the length coil.

Fig. 9. Variation of $Nu$ in the developing region of the length coil.

### 3. Influence of various coil parameters and development of correlation for estimation of average Nusselt number

In our earlier work (Jayakumar et al., 2008), the methodology for numerical estimation of heat transfer for a helically coiled heat exchanger has been successfully validated against experiments. There we considered only changes in flow rate of the stream, thus only changes in Dean number. In the current analysis, CFD simulations are carried out by varying coil parameters such as (i) pitch circle diameter, (ii) tube pitch and (iii) pipe diameter. In the next step we develop a unified correlation generated by CFD analysis, which will be applicable to wide range of helical configurations and Dean numbers. In this section, the results of analysis in which various coil parameters are changed are discussed. Subsequently correlations for prediction of Nusselt number are developed. Table 2 gives the details of the coil geometries used in the analysis carried out. Study has been carried out using the CFD package FLUENT 6.3 (3D, double precision). Each of the runs takes about 8 h on a Xenon 2.4 GHz computer with 2 GB RAM. Analysis has been carried out with both constant wall temperature and constant wall heat flux boundary conditions.

#### 3.1. Analysis with constant wall temperature boundary condition

This is one of the idealised boundary condition and is present in condensers and boilers. Details of discretisation scheme and boundary conditions are same as those provided in Section 2.1.

#### 2.2. Analysis with constant wall heat flux boundary condition

Analysis had also been carried out with a constant wall heat flux boundary condition. This situation is present in electrically heated pipes. The grid and the discretisation schemes used in this analysis were same as those presented in Section 2.1. The value of wall heat flux imposed in this analysis was equal to the average wall flux of the constant wall temperature case, presented in the previous section. Velocity and temperature of the inlet stream were also kept the same. These factors make the results from both the cases to be comparable with each other. Fig. 11 shows the velocity profile at various sections along the length of the coil for the case of constant wall heat flux BC. The values differ only marginally from the previous case.

Temperature profiles for this case are presented in Fig. 12. These profiles show a much different scenario as compared to Fig. 5. In contrast to the constant wall temperature case, heat transfer is uniform around the periphery up to $\theta = 50^\circ$. In addition, the percentage length of region with higher heat transfer rates is comparatively high for the case of constant wall heat flux BC. Three distinct regions of heat transfer, similar to that observed at $\theta = 150^\circ$ for the wall temperature case, is found to be occurring almost at the same angle for the constant wall flux case also. However, the percentage of the periphery with high heat transfer rates is significantly higher for the wall flux case. This behaviour results in higher values of Nusselt numbers as compared to the previous case. Temperature profile is fully developed by $\theta = 360^\circ$.
3.1.1. Influence of pitch circle diameter (PCD)

The coils with PCDs 100 mm, 200 mm, 300 mm and 400 mm were analysed. In all these analyses, the coil pitch and pipe diameter were kept at 30 mm and 20 mm respectively and the coils consisted of two turns.

The effect of PCD is to influence the centrifugal force on the moving fluid. This will in turn affect the secondary flows along the pipe cross-section. As the PCD is increased, the effect of coil curvature on flow decreases and hence centrifugal forces play a lesser role in flow characteristics. For the coil with PCD = 100 mm, the entrance effects are seen to be present up to an angle of 40°. While for PCDs 200 mm, 300 mm and 400 mm this change to 20°, 10° and 6° respectively. For the case of coil with PCD = 100 mm, the difference between Nusselt number at the inner and outer location in the fully developed heat transfer region is 200. As we move to coils of higher PCDs, this difference comes down and for a coil of PCD = 400 mm, it reduces to 134. Thus the effect of centrifugal force on heat transfer is evident.

A comparison of the average Nusselt number for different PCDs is shown in Table 3. The Nusselt number reported here are the average Nusselt numbers in the fully developed region (ref. Fig. 8). As the PCD is increased, the average value of Nu in the developed region comes down.

![Graphs showing variation of Nu around the circumference at various cross-section of the pipe](image-url)

Fig. 10. Variation of Nu around the circumference at various cross-section of the pipe (0° Inner, 90° bottom, 180° Outer and 270° top).
To correlate $Nu$ with pitch circle diameter of the coil, the dimensionless parameter curvature ratio $\delta (=r/R_c)$ is used. The correlation proposed is of the form,

$$Nu = C(\delta)^n$$

Fig. 15 gives a plot of $\log_e(\delta)$ vs. $\log_e(Nu)$. The data points are shown as circles and the equation is shown as a continuous line. The equation is found to give a good fit. From regression analysis using MATLAB®, Nusselt number can be correlated to curvature ratio as,

$$Nu = 265.65(\delta)^{0.11},$$

verifying the nature of the proposed correlation.

In order to numerically verify that the Nusselt number asymptotically approaches that for a straight pipe, a few more cases as given in Table 4 were analysed. It can be observed that as the PCD is made very large, the Nu approaches that for a straight pipe, as seen from the Fig. 16.
Fig. 11. Velocity (m s\(^{-1}\)) contours (a) at various planes along the length coil. (b) at selected planes along the length of the coil.

Table 2
Details of cases analysed.

<table>
<thead>
<tr>
<th>Case no.</th>
<th>PCD (mm)</th>
<th>Pitch (mm)</th>
<th>Pipe diameter (mm)</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>100</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>300</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>400</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>300</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>300</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>7</td>
<td>300</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>8</td>
<td>300</td>
<td>45</td>
<td>20</td>
</tr>
<tr>
<td>9</td>
<td>300</td>
<td>60</td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>300</td>
<td>45</td>
<td>30</td>
</tr>
<tr>
<td>11</td>
<td>300</td>
<td>45</td>
<td>40</td>
</tr>
</tbody>
</table>

Table 3
Average values of Nusselt number.

<table>
<thead>
<tr>
<th>PCD = 100 mm</th>
<th>PCD = 200 mm</th>
<th>PCD = 300 mm</th>
<th>PCD = 400 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nu(_{avg})</td>
<td>226.33</td>
<td>208.23</td>
<td>197.65</td>
</tr>
</tbody>
</table>

Fig. 12. Temperature (K) contours at (a) various planes along the length coil. (b) at selected planes along the length of the coil.

Table 4
Average values of Nusselt number for various PCDs.

<table>
<thead>
<tr>
<th>PCD = 1000 mm</th>
<th>PCD = 3000 mm</th>
<th>Straight pipe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nu(_{avg})</td>
<td>173.08</td>
<td>157.83</td>
</tr>
</tbody>
</table>
3.1.2. Influence of coil pitch (h)

In this analysis, a helical coil with a pipe of inner diameter \( (2r) \) 20 mm and pitch circle diameter (PCD) of 300 mm was considered. Analyses were carried out by changing the coil pitch. Coil with pitch of (i) 0 mm, (ii) 15 mm, (iii) 30 mm, (iv) 45 mm and (v) 60 mm were analysed.

When the coil pitch is zero, local Nusselt numbers at the top and bottom points on the periphery of a cross are almost the same. For this case, only centrifugal force but no torsional force is acting on the fluid. This causes symmetry about a horizontal plane passing through the centre of the pipe cross-section. Hence the values of local Nusselt numbers at the top and bottom points at a given cross-section are almost the same. As we increase the coil pitch, the torsional or rotational forces also comes into effect. This makes fluid streams to behave as shown in Fig. 6. The magnitude of difference between the local Nusselt numbers at the top and bottom at any given corresponding cross-section thus increases with increase in pitch. However, variation of local Nu for the coils with pitch of 45 and 60 mm are identical. Average values of Nusselt number in the fully developed region is given in Table 5.

---

Fig. 14. Variation of \( \text{Nu} \) along the periphery at various cross-section of the pipe (0° → inner, 90° → bottom, 180° → outer and 270° → top).
It is found that the $N_{u_{\text{avg}}}$ increases marginally with increase in pitch and almost insensitive to its further changes at higher pitches. The percentage increase, when the pitch is changed from 0 mm to 15 mm is about 1% and this value changes to 0.2% when the pitch is changed from 45 mm to 60 mm. For any engineering application, the tube pitch has to be higher than pipe diameter and in that range the changes in $N_{u_{\text{avg}}}$ due to changes in pitch are negligible. Hence the effect of coil pitch on overall heat transfer for design purposes need not be considered for most of the practical applications with helical coils. However, it has implications in heat transfer in the developing regions. The maximum difference in Nusselt number between the top and bottom locations is given in Table 6. This clearly shows the extent of oscillatory behaviour. Another observation is the shift of the symmetry in temperature

Table 5
Average values of Nusselt number for various coil pitches.

<table>
<thead>
<tr>
<th>$H$ (mm)</th>
<th>$N_{u_{\text{avg}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>189.24</td>
</tr>
<tr>
<td>15</td>
<td>191.08</td>
</tr>
<tr>
<td>30</td>
<td>191.75</td>
</tr>
<tr>
<td>45</td>
<td>192.27</td>
</tr>
<tr>
<td>60</td>
<td>192.55</td>
</tr>
</tbody>
</table>
Table 6
Difference in values of Nusselt number for various coil pitches.

<table>
<thead>
<tr>
<th></th>
<th>$H=0\text{ mm}$</th>
<th>$H=15\text{ mm}$</th>
<th>$H=30\text{ mm}$</th>
<th>$H=45\text{ mm}$</th>
<th>$H=60\text{ mm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. difference in values of $Nu_{loc}$ at top and bottom locations</td>
<td>0</td>
<td>7</td>
<td>12</td>
<td>18</td>
<td>26</td>
</tr>
</tbody>
</table>

Fig. 15. Regression analysis for variation of $Nu$ with PCD.

and velocity profiles with the change in coil pitch due to the torsion effects.

3.1.3. Influence of pipe diameter

In this analysis, the effect of pipe diameter on heat transfer in a helical coil is considered. The pipe diameters considered for analyses were, 10 mm, 20 mm, 30 mm and 40 mm. For all these cases, coil has a pitch of 45 mm and PCD of 300 mm and the coil consists of two turns.

For the coil with 10 mm diameter, Nusselt number in the top and bottom regions of the pipe are approximately equal. In the region of fully developed heat transfer, there is even uniform Nusselt number along the periphery of many planes. The values of local Nusselt number along the length is plotted in Fig. 17(a). When the pipe diameter is low, the secondary flows are weaker and hence mixing is lesser. This produces nearly the same heat transfer in the upper half cross-section in a given plane.

When the diameter of the coil is changed to 20 mm, in contrast to the case where $d=10$ mm, heat transfer at the outer side of the coil remain the highest for all of the sections. As expected, the length of pipe needed for the heat transfer to attain a fully developed state has increased as the pipe diameter is increased. In a similar way as the pipe diameter is increased to 40 mm, the secondary flows are substantial and this causes difference between the values of Nusselt numbers at the top and bottom locations for a given cross-section. The values of local Nusselt number is plotted in Fig. 17(b) and may be compared with Fig. 17(a). Table 7 gives the average values of Nusselt number in the fully developed region. Using MATLAB®, a regression analysis was carried out and the result showed a linear relationship between $Nu_{ave}$ and pipe diameter.

3.1.4. Correlation for estimation of Nusselt number

The correlation for Nusselt number already consists of pipe diameter in terms of Reynolds number and curvature ratio. Hence the correlation can be of the form,

$$Nu = CR_{e}^{a}Pr^{b}.$$

Fig. 16. Approaching of $Nu$ at high PCD to that for a straight pipe.

Fig. 17. (a) Local Nusselt number along the length for $d=10$ mm coil. (b) Local Nusselt number along the length for $d=40$ mm coil.
Table 7
Average values of Nusselt number for various pipe diameters.

<table>
<thead>
<tr>
<th>Pipe diameter (mm)</th>
<th>Nuavg</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>116.39</td>
</tr>
<tr>
<td>20</td>
<td>200.03</td>
</tr>
<tr>
<td>30</td>
<td>289.56</td>
</tr>
<tr>
<td>40</td>
<td>377.38</td>
</tr>
</tbody>
</table>

Table 8
Details of extra cases analysed for \( T_w \) BC.

<table>
<thead>
<tr>
<th>Case no.</th>
<th>PCD (mm)</th>
<th>Coil pitch (mm)</th>
<th>Pipe diameter (mm)</th>
<th>Inlet velocity ( (m\ s^{-1}) )</th>
<th>Inlet temperature (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>150</td>
<td>20</td>
<td>12</td>
<td>0.8</td>
<td>340</td>
</tr>
<tr>
<td>14</td>
<td>150</td>
<td>20</td>
<td>12</td>
<td>1.0</td>
<td>340</td>
</tr>
<tr>
<td>15</td>
<td>150</td>
<td>20</td>
<td>12</td>
<td>1.2</td>
<td>340</td>
</tr>
<tr>
<td>16</td>
<td>250</td>
<td>40</td>
<td>25</td>
<td>0.6</td>
<td>350</td>
</tr>
<tr>
<td>17</td>
<td>250</td>
<td>40</td>
<td>25</td>
<td>1.0</td>
<td>350</td>
</tr>
<tr>
<td>18</td>
<td>250</td>
<td>40</td>
<td>25</td>
<td>1.4</td>
<td>350</td>
</tr>
<tr>
<td>19</td>
<td>300</td>
<td>30</td>
<td>20</td>
<td>0.6</td>
<td>330</td>
</tr>
<tr>
<td>20</td>
<td>400</td>
<td>30</td>
<td>20</td>
<td>0.5</td>
<td>330</td>
</tr>
</tbody>
</table>

Fig. 19 gives a comparison of the Nusselt numbers predicted by the correlation so developed with the earlier correlations. Fig. 18 gives the comparison of the data points and the new correlation.

3.2. Analysis with constant wall heat flux boundary condition

In this section, results of analysis with constant wall heat flux boundary condition are presented. This boundary condition is applicable to heat flux-controlled surfaces such as electrically heated pipes. The geometry of the coils analysed is listed in Table 2. The schemes of discretisation and boundary conditions used in these analyses are provided in Section 2.2. Analyses were carried out for a specified wall heat flux of \(-150 \text{ kW m}^{-2}\).

Multiple regression analysis based on the data generated from above case studies has been done using MATLAB\textsuperscript{®}. The correlation so developed for estimation of Nusselt number is given in Eq. (12). The range of these parameters are (i) \(14,000 < Re < 70,000\); (ii) \(3000 < De < 22,000\); (iii) \(3.0 < Pr < 5.0\); and (iv) \(0.05 < \delta < 0.2\).

\[ Nu = 0.116Re^{0.71}Pr^{0.4} \delta^{0.11}. \]  

\[ (12) \]

Fig. 18 shows the comparison of the data points and the developed correlation.

Fig. 19 gives a comparison of the Nusselt numbers predicted by Eq. (12) with (i) Rogers and Mayhew (1964) and (ii) Mori and Nakayama (1967b). It is found that present correlation is fairly in agreement with Nusselt number predicted by correlations by Rogers and Mayhew (1964) and Mori and Nakayama (1967b). The earlier correlations are found to be under predicting the Nusselt number. This is due to the approximations used by the authors in data reduction and conservative nature of their approach.

Table 9
Details of extra cases analysed for \( q_w' \) B.C.

<table>
<thead>
<tr>
<th>Case no.</th>
<th>PCD (mm)</th>
<th>Coil pitch (mm)</th>
<th>Pipe diameter (mm)</th>
<th>Inlet velocity ( (m\ s^{-1}) )</th>
<th>Heat flux ( q_w' ) ( (\text{W m}^{-2}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>150</td>
<td>20</td>
<td>12</td>
<td>0.8</td>
<td>100.0</td>
</tr>
<tr>
<td>14</td>
<td>150</td>
<td>20</td>
<td>12</td>
<td>1.0</td>
<td>150.0</td>
</tr>
<tr>
<td>15</td>
<td>150</td>
<td>20</td>
<td>12</td>
<td>1.2</td>
<td>150.0</td>
</tr>
<tr>
<td>16</td>
<td>250</td>
<td>40</td>
<td>25</td>
<td>0.6</td>
<td>120.0</td>
</tr>
<tr>
<td>17</td>
<td>250</td>
<td>40</td>
<td>25</td>
<td>1.0</td>
<td>150.0</td>
</tr>
<tr>
<td>18</td>
<td>250</td>
<td>40</td>
<td>25</td>
<td>1.4</td>
<td>180.0</td>
</tr>
<tr>
<td>19</td>
<td>300</td>
<td>30</td>
<td>20</td>
<td>0.6</td>
<td>100.0</td>
</tr>
<tr>
<td>20</td>
<td>400</td>
<td>30</td>
<td>20</td>
<td>0.5</td>
<td>120.0</td>
</tr>
</tbody>
</table>
an error more than 20%. Thus the earlier correlations are found to be under-predicting the Nusselt number.

A comparison of the Nusselt numbers generated from the correlations given by Eqs. (12) and (13) is presented in Fig. 22. It can be seen that both the correlations give almost the same value of Nusselt number at lower values of Reynolds number. However, they show marginal difference when \( Re > 50,000 \).

3.3. Conjugate heat transfer boundary condition

In the paper by Jayakumar et al. (2008), a correlation was developed for estimation of Nusselt number considering conjugate heat transfer. It is also found that the percentage difference between conjugate heat transfer and constant wall flux boundary conditions is about 8. Thus use of heat flux boundary condition is a good engineering approximation for estimation of heat transfer for the case of conjugate heat transfer. Since the effort required for analysing heat transfer with conjugate heat transfer may not be worth from design point of view, results of constant wall heat flux boundary condition may be used for the conjugate case as well.

3.4. Variation of local nusselt number

It has been shown in Section 2 that the heat transfer and hence the Nusselt number is not uniform along the periphery at any given cross-section of the helical pipe. In this section an expression is developed to estimate the Nusselt number at various points along the periphery in the fully developed region.

3.4.1. Constant wall temperature boundary condition

Fig. 23 shows plot of ratio of local Nusselt number to the average Nusselt number (\( \frac{N_{u_{loc}}}{N_{u_{avg}}} \)) as a function of angle (\( \phi \)) for the constant wall heat flux boundary condition.
different cases presented in Tables 2 and 8. The angle \( \phi \) (in degrees) is measured anti-clockwise, starting from the inner side of the coil. The \( \text{Nu}_{\text{avg}} \) is Nusselt number obtained using Eq. (12). It is observed that except in the regions close to the inner side of the pipe, the distribution of \( \text{Nu}_{\text{loc}} \) is almost independent of the coil geometry and Dean number. Based on these data the following correlation is developed for the prediction of local Nusselt number as a function of average Nusselt number and angular location.

\[
\text{Nu}_{\text{loc}} = \text{Nu}_{\text{avg}}(-2.411 - 0.5\phi^2 + 8.692 - 0.3\phi + 0.4215) \tag{14}
\]

3.4.2. Constant wall heat flux boundary condition

A similar exercise has been carried out to correlate the variation of local Nusselt number for the constant wall heat flux boundary condition. The plot of \( \text{Nu}_{\text{loc}}/\text{Nu}_{\text{avg}} \) as a function of \( \phi \) for the different cases presented in Tables 2 and 8 is shown in Fig. 24. The average value of Nusselt number (\( \text{Nu}_{\text{avg}} \)) is obtained using Eq. (13). The following correlation can be used for the prediction of local Nusselt number as a function of average Nusselt number and angular location.

\[
\text{Nu}_{\text{loc}} = \text{Nu}_{\text{avg}}(-2.331 - 0.5\phi^2 + 8.424 - 0.3\phi + 0.4576) \tag{15}
\]

4. Conclusion

1. Characteristics of heat transfer under turbulent flow of single-phase water through helical coils of vertical axis are presented in the article. Necessary Python codes, which run in the framework of AnuVi visualisation package, have been developed for accurate estimation of Nusselt number at any point on the heat transfer surface. The developmental work also includes generation of various C++ and MATLAB codes.

2. Analysis has been carried out both for the constant wall temperature and constant wall heat flux boundary conditions. Fluid particles are found to undergo oscillatory motion inside the pipe and this causes fluctuations in heat transfer rates.

3. Nusselt numbers at various points along the length of the pipe was estimated. Nusselt number on the outer side of the coil is found to be the highest among all other points at a specified cross-section, while that at the inner side of the coil is the lowest. Velocity profiles for the two boundary conditions were found to be matching, while the temperature profiles are different.

4. A number of numerical experiments have been carried out to study influence of coil parameters, viz., pitch circle diameter, coil pitch and pipe diameter on heat transfer. The coil pitch is found to have significance only in the developing section of heat transfer. The torsional forces induced by the pitch causes oscillations in the Nusselt number. However, the average Nusselt number is not affected by the coil pitch. Hence it is established that any correlation for estimation of Nusselt number for applications to helical coils shall include the coil parameters pitch circle radius and pipe diameter apart form the flow parameters and physical properties.

5. After establishing the parametric influence, a correlation has been developed for estimation of average Nusselt number. This correlation is compared with those available in the literature and the deviations are within reasonable limits. It is also observed that these correlations are applicable for either of the boundary conditions. For most of the engineering applications, the correlations are applicable for conjugate heat transfer as well.

6. In the fully developed section, ratio \( \text{Nu}_{\text{loc}}/\text{Nu}_{\text{avg}} \) is almost independent of coil parameters and Dean number. Correlations have been developed for prediction of local values of Nusselt number as a function of the average Nusselt number and the angular position of the point along the circumference.

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References


