











# External Flow Correlations (Average, Isothermal Surface)

## Flat Plate Correlations

Flow Conditions	Average Nusselt Number	Restrictions
Laminar	$\overline{Nu}_L = 0.664 Re_L^{1/2} Pr^{1/3}$	$Pr \geq 0.6$
Turbulent	$\overline{Nu}_L = (0.037 Re_L^{4/5} - A) Pr^{1/3}$ where $A = 0.037 Re_{x,c}^{4/5} - 0.664 Re_{x,c}^{1/2}$	$0.6 \leq Pr \leq 60$ $Re_{x,c} \leq Re_L \leq 10^8$

Note: All fluid properties are evaluated at film temperature for flat plate correlations.

## Cylinders in Cross Flow

Cylinder Cross Section	Reynolds Number Range	Average Nusselt Number	Restrictions
	0.4-4	$\overline{Nu}_D = 0.989 Re_D^{0.330} Pr^{1/3}$	$Pr \geq 0.7$
	4-40	$\overline{Nu}_D = 0.911 Re_D^{0.385} Pr^{1/3}$	$Pr \geq 0.7$
	40-4,000	$\overline{Nu}_D = 0.683 Re_D^{0.466} Pr^{1/3}$	$Pr \geq 0.7$
	4,000-40,000	$\overline{Nu}_D = 0.193 Re_D^{0.618} Pr^{1/3}$	$Pr \geq 0.7$
	40,000-400,000	$\overline{Nu}_D = 0.027 Re_D^{0.805} Pr^{1/3}$	$Pr \geq 0.7$
	6,000-60,000	$\overline{Nu}_L = 0.304 Re_D^{0.59} Pr^{1/3}$	gas flow
	5,000-60,000	$\overline{Nu}_D = 0.158 Re_D^{0.66} Pr^{1/3}$	gas flow
	5,200-20,400	$\overline{Nu}_D = 0.164 Re_D^{0.638} Pr^{1/3}$	gas flow
	20,400-105,000	$\overline{Nu}_D = 0.039 Re_D^{0.78} Pr^{1/3}$	gas flow
	4,500-90,700	$\overline{Nu}_D = 0.150 Re_D^{0.638} Pr^{1/3}$	gas flow

Note: All fluid properties are evaluated at film temperature for cylinder in cross flow correlations.

### Alternative Correlations for Circular Cylinders in Cross Flow:

- The Zukauskas correlation (7.53) and the Churchill and Bernstein correlation (7.54) may also be used

## Freely Falling Liquid Drops

Average Nusselt Number

$$\overline{Nu}_D = 2 + 0.6 Re_D^{1/2} Pr^{1/3}$$

Note: All fluid properties are evaluated at  $T_\infty$  for the falling drop correlation.

## Flow Around a Sphere

Average Nusselt Number	Restrictions
$\overline{Nu}_D = 2 + (0.4 Re_D^{1/2} + 0.06 Re_D^{2/3}) Pr^{0.4} \left( \frac{\mu}{\mu_s} \right)^{1/4}$	$0.71 \leq Pr \leq 380$ $3.5 \leq Re_D \leq 7.6 \times 10^4$ $1.0 \leq (\mu / \mu_s) \leq 3.2$

Note: For flow around a sphere, all fluid properties, except  $\mu_s$ , are evaluated at  $T_\infty$ .  $\mu_s$  is evaluated at  $T_s$ .

# Internal Flow Correlations (Local, Fully Developed Flow)











Note: For all local correlations, fluid properties are evaluated at  $T_m$ .

For average correlations, fluid properties are evaluated at the average of inlet and outlet  $T_m$ .

If the tube is much longer than the thermal entry length, average correlation  $\approx$  local correlation.

## Laminar Flow in Circular and Noncircular Tubes

$$Nu_D \equiv \frac{hD_h}{k}$$

Cross Section	$\frac{b}{a}$	Uniform Heat Flux	Uniform Surface Temperature	$f Re_{D_h}$
	—	4.36	3.66	64
	1.0	3.61	2.98	57
	1.43	3.73	3.08	59
	2.0	4.12	3.39	62
	3.0	4.79	3.96	69
	4.0	5.33	4.44	73
	8.0	6.49	5.60	82
	$\infty$	8.23	7.54	96
	$\infty$	5.39	4.86	96
	—	3.11	2.49	53

## Turbulent Flow in Circular Tubes

Local Nusselt Number	Restrictions
$Nu_D = 0.023 Re_D^{4/5} Pr^n$ $n = 0.40$ for $T_s > T_m$ $n = 0.30$ for $T_s < T_m$	$0.6 \leq Pr \leq 160$ $Re_D \geq 10,000$ $(L/D) \geq 10$

## Turbulent Flow in Noncircular Tubes

For turbulent flow in noncircular tubes,  $D$  in the table above may be replaced by  $D_h = 4A_c / P$

## Alternative Correlations for Turbulent Flow in Circular Tubes:

- The Sieder and Tate Correlation (8.61) is recommended for flows with large property variations
- Another alternate correlation that is more complex but more accurate is provided by Gnielinski (8.62).

## Liquid Metals, Turbulent Flow, Constant $T_s$

Local Nusselt Number	Restrictions
$Nu_D = 5.0 + 0.025 Pe_D^{0.8}$ $Pe_D = Re_D Pr$	$Pe_D \geq 100$

Note: Only use the correlation in the box directly above for liquid metals. The other correlations on this page are not applicable to liquid metals.

# Combined Internal/External Flow Correlations (Average)

Tube banks and packed beds have characteristics of both internal and external flow. The flow is internal in that the fluid flows inside the tube bank/packed bed, exhibits exponential temperature profiles of the mean temperature, and has heat transfer governed by a log mean temperature difference. The flow is external in that it flows over tubes/packed bed particles and that the characteristic dimension in the Reynolds number is based on tube/particle diameter.

## Tube Bank Correlation

Average Nusselt Number	Restrictions
$\overline{Nu}_D = C Re_{D,\max}^m Pr^{0.36} \left( \frac{Pr}{Pr_s} \right)^{1/4}$	$N_L \geq 20$ $0.7 \leq Pr \leq 500$ $10 \leq Re_{D,\max} \leq 2 \times 10^6$

Note: For tube banks with fewer than 20 rows, multiply the average Nusselt number from the table at left by the correction factor  $C_2$  in Table 7.6. This correction is valid if  $Re_{D,\max}$  is  $> 1,000$ .

Configuration	$Re_{D,\max}$	$C$	$m$
Aligned	$10-10^2$	0.80	0.40
Staggered	$10-10^2$	0.90	0.40
Aligned	$10^2-10^3$	Approximate as a single (isolated) cylinder	
Staggered	$10^2-10^3$		
Aligned ( $S_T/S_L > 0.7$ ) <sup>a</sup>	$10^3-2 \times 10^5$	0.27	0.63
Staggered ( $S_T/S_L < 2$ )	$10^3-2 \times 10^5$	$0.35(S_T/S_L)^{1/5}$	0.60
Staggered ( $S_T/S_L > 2$ )	$10^3-2 \times 10^5$	0.40	0.60
Aligned	$2 \times 10^5-2 \times 10^6$	0.021	0.84
Staggered	$2 \times 10^5-2 \times 10^6$	0.022	0.84

<sup>a</sup>For  $S_T/S_L < 0.7$ , heat transfer is inefficient and aligned tubes should not be used.

## Packed Bed Correlation

Average Nusselt Number	Restrictions
$\varepsilon \overline{j}_H = \varepsilon \overline{j}_M = 2.06 Re_D^{-0.575}$ <p>where</p> $\overline{j}_H = \frac{\overline{h}}{\rho V c_p} Pr^{2/3}$ $\overline{j}_M = \frac{\overline{h}_m}{V} Sc^{2/3}$	$Pr \text{ (or } Sc) \approx 0.7$ $90 \leq Re_D \leq 4,000$

# External Free Convection Correlations (Average, Isothermal)

Evaluate all fluid properties at the film temperature  $T_f = (T_\infty + T_s) / 2$ .

## Vertical Plate, Vertical Cylinder, Top Side of Inclined Cold Plate, Bottom Side of Inclined Hot Plate

Average Nusselt Number	Restrictions
$\overline{Nu}_L = \left\{ 0.825 + \frac{0.387 Ra_L^{1/6}}{\left[ 1 + (0.492 / Pr)^{9/16} \right]^{8/27}} \right\}^2$	<p><u>Vertical plate</u>: no restrictions</p> <p><u>Vertical cylinder</u>: <math>\frac{D}{L} \geq \frac{35}{Gr_L^{1/4}}</math></p> <p><u>Top Surface of Inclined Cold Plate / Bottom Surface of Inclined Hot Plate</u>:</p> <ul style="list-style-type: none"> <li>• Replace <math>g</math> with <math>g \cos \theta</math> in <math>Ra_L</math></li> <li>• Valid for <math>0 \leq \theta \leq 60</math></li> </ul>

### Alternative Correlation for Vertical Plate:

▪ Equation (9.27) is slightly more accurate for laminar flow.

## Horizontal Plate

Orientation	Average Nusselt Number	Restrictions
Upper surface of hot plate or lower surface of cold plate	$\overline{Nu}_L = 0.54 Ra_L^{1/4}$	$10^4 \leq Ra_L \leq 10^7, Pr \geq 0.7$
	$\overline{Nu}_L = 0.15 Ra_L^{1/3}$	$10^7 \leq Ra_L \leq 10^{11}, \text{ all } Pr$
Lower surface of hot plate or upper surface of cold plate	$\overline{Nu}_L = 0.52 Ra_L^{1/5}$	$10^4 \leq Ra_L \leq 10^9, Pr \geq 0.7$

$$L \equiv \frac{A_s}{P}$$

## Curved Shapes

Shape	Average Nusselt Number	Restrictions
Long Horizontal Cylinder	$\overline{Nu}_D = \left\{ 0.60 + \frac{0.387 Ra_D^{1/6}}{\left[ 1 + (0.559 / Pr)^{9/16} \right]^{8/27}} \right\}^2$	$Ra_D \leq 10^{12}$
Sphere	$\overline{Nu}_D = 2 + \frac{0.589 Ra_D^{1/4}}{\left[ 1 + (0.469 / Pr)^{9/16} \right]^{4/9}}$	$Pr \geq 0.7$ $Ra_D \leq 10^{11}$

### Alternative Correlation for Long Horizontal Cylinder:

▪ The Morgan correlation (9.33) may also be used.

# Internal Free Convection Correlations

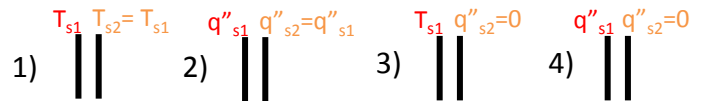
## Vertical Parallel Plate Channels (Developing and Fully Developed)

Boundary Condition	Nusselt Number	Rayleigh Number	Getting q and q'' from Nu	Temperature to evaluate fluid properties in Ra
isothermal (T <sub>s</sub> known on one or both plates)	<u>Average Nu over whole plate</u> $\overline{Nu}_S = \left[ \frac{C_1}{(Ra_S S/L)^2} + \frac{C_2}{(Ra_S S/L)^{1/2}} \right]^{-1/2}$	$Ra_S = \frac{g\beta(T_s - T_\infty)S^3}{\alpha\nu}$	$\overline{Nu}_S = \left( \frac{q/A}{T_s - T_\infty} \right) \frac{S}{k}$	$\overline{T} = \frac{T_s + T_\infty}{2}$
isoflux (q'' known on one or both plates)	<u>Local Nu at x=L</u> $Nu_{S,L} = \left[ \frac{C_1}{Ra_{S,L}^* S/L} + \frac{C_2}{(Ra_{S,L}^* S/L)^{2/5}} \right]^{-1/2}$	$Ra_{S,L}^* = \frac{g\beta q'' S^4}{k\alpha\nu}$	$Nu_{S,L} = \left( \frac{q''}{T_{s,L} - T_\infty} \right) \frac{S}{k}$	$\overline{T} = \frac{T_{s,L} + T_\infty}{2}$

S = plate spacing; T<sub>∞</sub>=inlet temperature (same as ambient); T<sub>s,L</sub>=surface temperature at x=L

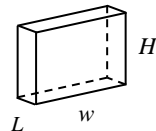
Surface Condition	C <sub>1</sub>	C <sub>2</sub>
1) Symmetric <u>isothermal</u> plates (T <sub>s,1</sub> = T <sub>s,2</sub> )	576	2.87
2) Symmetric <u>isoflux</u> plates (q'' <sub>s,1</sub> = q'' <sub>s,2</sub> )	48	2.51
3) <u>Isothermal/adiabatic</u> plates (T <sub>s,1</sub> , q'' <sub>s,2</sub> = 0)	144	2.87
4) <u>Isoflux/adiabatic</u> plates (q'' <sub>s,1</sub> , q'' <sub>s,2</sub> = 0)	24	2.51

C<sub>1</sub> and C<sub>2</sub> are given for four different sets of surface boundary conditions. Use the isothermal equation for conditions 1 and 3; isoflux equation for conditions 2 and 4.



## Vertical Rectangular Cavity

Average Nusselt Number	Restrictions
$\overline{Nu}_L = 0.18 \left( \frac{\text{Pr}}{\text{Pr} + 0.2} Ra_L \right)^{0.29}$	$1 \leq (H/L) \leq 2$ $10^{-3} \leq \text{Pr} \leq 10^5$ $10^3 \leq \frac{Ra_L \text{Pr}}{0.2 + \text{Pr}}$
$\overline{Nu}_L = 0.22 \left( \frac{\text{Pr}}{\text{Pr} + 0.2} Ra_L \right)^{0.28} \left( \frac{H}{L} \right)^{-1/4}$	$2 \leq (H/L) \leq 10$ $\text{Pr} \leq 10^5$ $10^3 \leq Ra_L \leq 10^{10}$
$\overline{Nu}_L = 0.42 Ra_L^{1/4} \text{Pr}^{0.012} \left( \frac{H}{L} \right)^{-0.3}$	$10 \leq (H/L) \leq 40$ $1 \leq \text{Pr} \leq 2 \times 10^4$ $10^4 \leq Ra_L \leq 10^7$



### Alternative Correlation for Vertical Rectangular Cavity:

Eq. (9.53) covers a wide range of aspect ratios but is more restrictive on Ra and Pr

### Horizontal Cavity Heated From Below

Average Nusselt Number	Restrictions
$\overline{Nu}_L = 0.069 Ra_L^{1/3} \text{Pr}^{0.074}$	$3 \times 10^5 \leq Ra_L \leq 7 \times 10^9$

For cavity correlations, evaluate all fluid properties at the average surface temperature  $\overline{T} = (T_1 + T_2) / 2$ . L is the distance between hot and cold walls.

### Correlations for Inclined/Tilted Geometries:

- Inclined parallel plate channels: (9.47)
- Tilted rectangular cavities: (9.54)-(9.57)

### Correlations for Curved Geometries:

- Space between concentric horizontal cylinders: (9.58)
- Space between concentric spheres: (9.61)

# Boiling and Condensation

## Nucleate Pool Boiling

$$q_s'' = \mu_l h_{fg} \left[ \frac{g(\rho_l - \rho_v)}{\sigma} \right]^{1/2} \left( \frac{c_{p,l} \Delta T_e}{C_{s,f} h_{fg} Pr_l^n} \right)^3$$

Evaluate liquid and vapor properties at  $T_{sat}$ .

Surface-Fluid Combination	$C_{s,f}$	$n$
Water-copper		
Scored	0.0068	1.0
Polished	0.0128	1.0
Water-stainless steel		
Chemically etched	0.0133	1.0
Mechanically polished	0.0132	1.0
Ground and polished	0.0080	1.0
Water-brass	0.0060	1.0
Water-nickel	0.006	1.0
Water-platinum	0.0130	1.0
<i>n</i> -Pentane-copper		
Polished	0.0154	1.7
Lapped	0.0049	1.7
Benzene-chromium	0.0101	1.7
Ethyl alcohol-chromium	0.0027	1.7

## Critical Heat Flux

$$q_{max}'' = Ch_{fg} \rho_v \left[ \frac{\sigma g(\rho_l - \rho_v)}{\rho_v^2} \right]^{1/4}$$

Evaluate liquid and vapor properties at  $T_{sat}$ .

$C = 0.149$  for large horizontal plates.

$C = 0.131$  for large horizontal cylinders, spheres, and many large finite heated surfaces.

## Film Boiling

$$\overline{Nu}_D = \frac{\bar{h}_{conv} D}{k_v} = C \left[ \frac{g(\rho_l - \rho_v) h'_{fg} D^3}{\nu_v k_v (T_s - T_{sat})} \right]^{1/4}$$

$$h'_{fg} = h_{fg} + 0.80 c_{p,v} (T_s - T_{sat})$$

Evaluate vapor properties at  $T_f = (T_{sat} + T_s) / 2$ .

Evaluate  $\rho_l$  and  $h_{fg}$  at  $T_{sat}$ .

$C = 0.67$  for spheres.  $C = 0.62$  for horizontal cylinders.

Radiation should be considered for  $T_s > 300^\circ\text{C}$

See Eqs. (10.9)-(10.11)

## Correlations for Flow Boiling:

- External forced convection boiling: (10.12)-(10.14)
- Two-phase flow: (10.15)-(10.16)

## For all condensation correlations below:

Evaluate liquid properties at  $T_f = (T_{sat} + T_s) / 2$ .

Evaluate  $\rho_v$  and  $h_{fg}$  at  $T_{sat}$ .

## Laminar Film Condensation, Vertical Flat Plate

$$\bar{h}_L = 0.943 \left[ \frac{\rho_l g(\rho_l - \rho_v) h'_{fg} k_l^3}{\mu_l (T_{sat} - T_s) L} \right]^{1/4}$$

$$h'_{fg} = h_{fg} + 0.68 c_{p,l} (T_{sat} - T_s)$$

## Laminar, Transition, and Turbulent Film Condensation, Vertical Flat Plate (for $\rho_l \gg \rho_v$ ):

- Calculate the parameter  $P$  using (10.42), then solve for  $\bar{h}_L$  using the appropriate correlation from (10.43)-(10.45)

## Film Condensation, Vertical Tube:

- Vertical flat plate expressions can be used if  $\delta(L) \ll D/2$ . Evaluate  $\delta(L)$  using (10.26).

## Laminar Film Condensation, Sphere and Tube

$$\bar{h}_D = C \left[ \frac{\rho_l g(\rho_l - \rho_v) h'_{fg} k_l^3}{\mu_l (T_{sat} - T_s) D} \right]^{1/4}$$

$$h'_{fg} = h_{fg} + 0.68 c_{p,l} (T_{sat} - T_s)$$

$C = 0.826$  for spheres.

$C = 0.729$  for horizontal tubes.

## Laminar Film Condensation, Vertical Tier of N Tubes:

- Average heat transfer coefficient of each tube: Eq. (10.49).

## Inner Surface of Horizontal Tube

Average Nusselt Number	Restrictions
$\bar{h}_D = 0.555 \left[ \frac{\rho_l g(\rho_l - \rho_v) h'_{fg} k_l^3}{\mu_l (T_{sat} - T_s) D} \right]^{1/4}$ $h'_{fg} = h_{fg} + 0.375 c_{p,l} (T_{sat} - T_s)$	$\left( \frac{\rho_v u_{m,v} D}{\mu_v} \right)_i < 35,000$
Eq. (10.51)	$\left( \frac{\rho_v u_{m,v} D}{\mu_v} \right)_i \geq 35,000$

## Dropwise Condensation

Average Nusselt Number	Restrictions
$\bar{h}_{dc} = 51,104 + 2044 T_{sat} (\text{°C})$	$22^\circ\text{C} \leq T_{sat} \leq 100^\circ\text{C}$
$\bar{h}_{dc} = 255,510$	$T_{sat} \geq 100^\circ\text{C}$